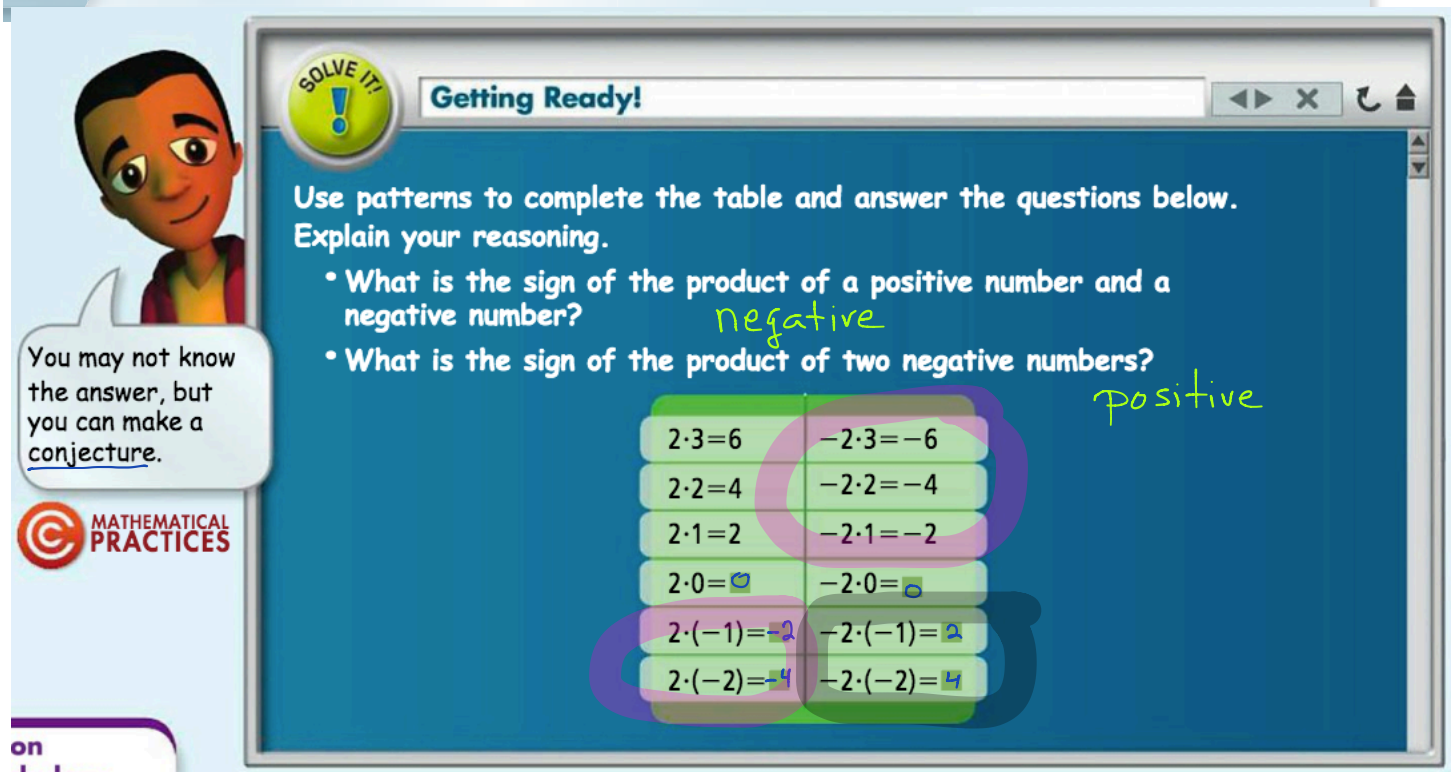
1-6 Multiplying and Dividing Real Numbers



The rules for multiplying real numbers are related to the properties of real numbers and the definitions of operations.

(3)(5)

5+5+5

15

(3)(-5)

(-5)+(-5)+(-5)

−15

(-3)(5)

(-3)+(-3)+(-3)+(-3)+(-3)

−15

(-3)(-5)

(-1)(3)(-1)(5)

[(-1)(-1)][(3)(5)]

1 · 15

15

KEY CONCEPT: MULTIPLYING REAL NUMBERS

The product of two real numbers with different signs is *negative*.

Examples: $2(-3) = -6$ $-2(3) = -6$

The product of two real numbers with the same sign is *positive*.

Examples: $2(3) = 6$ $(-2)(-3) = 6$

PROBLEM 1: MULTIPLYING REAL NUMBERS

What is each product?

a) $12(-8)$ $\begin{array}{r}{}^{1}12\\ \times\ 8\\ \hline 96\end{array}$

-96

b). $24(0.5)$ $\begin{array}{r}{}^{2}24\\ \times\ .5\\ \hline 12.0\end{array}$

$+12$

c). $-\frac{3}{4}\cdot\frac{1}{2}$

$-\frac{3}{8}$

d). $(-3)^2$

$(-3)(-3)$

$+9$

$a^2=b$

$\sqrt{b}=a$

$\sqrt{9}=\pm 3$

$c^2=a^2+b^2$

$c^2=(3)^2+(4)^2$

$c^2=9+16$

$\sqrt{c^2}=\sqrt{25}$

$c=+5$ or -5

e) $6(-15)$ $\begin{array}{r}{}^{3}15\\ \times\ 6\\ \hline 90\end{array}$

-90

f). $12(0.2)$ $\begin{array}{r}12\\ \times .2\\ \hline 2.4\end{array}$

$+2.4$

g). $\frac{7}{10}\left(-\frac{1}{2}\right)$

$-\frac{7}{20}$

h). $(-4)^2$

$(-4)(-4)$

$+16$

Notice that $(-3)^2$ in part (d) of Problem 1. Recall from Lesson 1-3 that a is a square root of b if $a^2=b$. So -3 is a square root of 9. A negative square root is represented by $-\sqrt{\ }$. Every positive real number has a positive and a negative square root. The symbol $\pm$ in front of the radical indicates both square roots.

PROBLEM 2: SIMPLIFYING SQUARE ROOT EXPRESSIONS

What is the simplified form of each expression?

a) $-\sqrt{25}$

-5

b). $\pm\sqrt{\frac{4}{49}}$

$\pm\frac{2}{7}$

$\frac{2}{7}$ and $-\frac{2}{7}$

c). $\sqrt{81}$

9

d). $\sqrt{-16}$

NOT A REAL NUMBER!

$-\sqrt{16}$

-4

e) $\sqrt{64}$

8

f). $\pm\sqrt{4}$

± 2

g). $-\sqrt{121}$

-11

h). $\pm\sqrt{\frac{1}{36}}$

$\pm\frac{1}{6}$

KEY CONCEPT: DIVIDING REAL NUMBERS

The quotient of two real numbers with <u>different signs</u> is *negative*.

Examples: $-20\div 5=-4$ $20\div(-5)=-4$

The quotient of two real numbers with the <u>same sign</u> is *positive.*

Examples: $20\div 5=4$ $-20\div(-5)=4$

PROBLEM 3: DIVIDING REAL NUMBERS

a) A sky diver's elevation changes by -3600 ft in 4 min after the parachute opens. What is the average change in the sky diver's elevation each minute?

$\frac{-3600\text{ ft}}{+4\text{ min}} = -900\text{ ft/min}$

$4\overline{)3600.}$ = 900

The elevation is decreasing 900 feet per minute.

b) You make five withdrawals of equal amounts from your bank account. The total amount you withdraw is \$520. What is the change in your account balance each time you make a withdrawal?

$\frac{-\$520}{+5\text{ withdrawals}} = -\$104\text{/withdrawal}$

$5\overline{)520}$ = 104

Account decreases \$104 per withdrawal.

PROPERTY: INVERSE PROPERTY OF MULTIPLICATION

For every nonzero real number *a*, there is a ***multiplicative inverse*** $\frac{1}{a}$ such that $a\left(\frac{1}{a}\right)=1$

Example: The multiplicative inverse of -4 is $-\frac{1}{4}$ because $-4\left(-\frac{1}{4}\right)=1$.

The ***reciprocal*** *of a nonzero real number of the form* $\frac{a}{b}$ *is* $\frac{b}{a}$. The product of a number and its reciprocal is 1, so the reciprocal of a number is its multiplicative inverse. This allows are rule for dividing fractions.

KEY CONCEPT: DIVIDING FRACTIONS

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

When dividing with fractions, rewrite the problem as the multiplication by the reciprocal of the denominator.

PROBLEM 4: DIVIDING FRACTIONS

Simplify.

a) $\frac{-\frac{3}{4}}{-\frac{2}{3}}$

$-\frac{3}{4} \div \left(-\frac{2}{3}\right)$

$-\frac{3}{4} \cdot \left(-\frac{3}{2}\right)$

$\frac{9}{8}$

b) $\frac{\frac{4}{5}}{-\frac{5}{3}}$

$\frac{4}{5} \div \left(-\frac{5}{3}\right)$

$\frac{4}{5} \cdot \left(-\frac{3}{5}\right)$

$-\frac{12}{25}$

c) $-7 \div \frac{7}{3}$

$\frac{7}{7}$

$-\frac{7}{1} \cdot \frac{3}{7} \Rightarrow -\frac{3}{1} = -3$

$-\frac{21}{7}$

$-\frac{3}{1}$

-3

d) $\frac{3}{8} \div \frac{3}{1}$

$\frac{3}{8} \cdot \frac{1}{3} \Rightarrow \frac{3}{24}$

$\frac{1}{8}$

e) $\frac{\frac{2}{3}}{-\frac{1}{4}}$

$\frac{2}{3} \div \left(-\frac{1}{4}\right)$

$\frac{2}{3} \cdot \left(-\frac{4}{1}\right)$

$-\frac{8}{3}$

f) $20 \div \frac{1}{4}$

$\frac{20}{1} \cdot \frac{4}{1}$

$\frac{80}{1}$

80

g) $\frac{2}{7} \div \left(-\frac{20}{21}\right)$

$\frac{2}{7} \cdot \left(-\frac{21}{20}\right) \Rightarrow -\frac{42}{140}$

$-\frac{3}{10}$